Operation Analysis and Transfer Function Derivation of Three-Phase Unfolding Inverter for Renewable Energy

PEEC Laboratry

2026 Joint University Student Workshop

2026. 05.14

Presenter : Su Ho Park

Author : Su Ho Park, Eun Seop Kim, Hag Wone Kim†

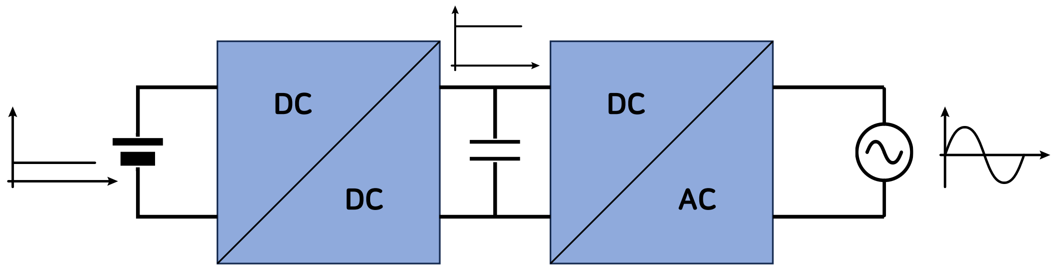
E-mail : parksuho418@a.ut.ac.kr

Table of Contents

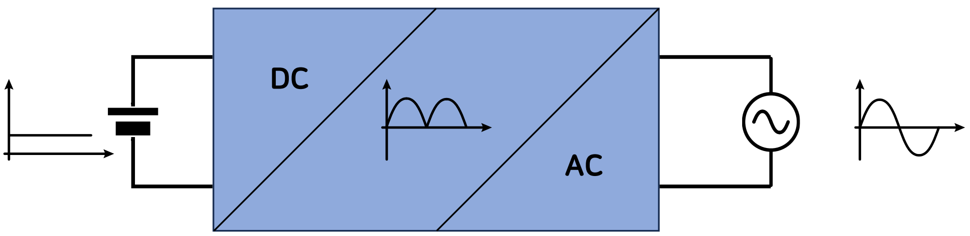
PEEC Laboratry

Unfolding Circuit

- Basic Concept of the Unfolding Circuit



<General DC-AC conversion>



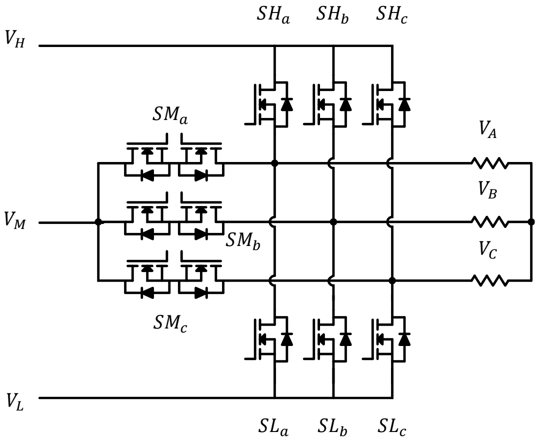
<DC-AC conversion using an unfolding circuit>

- General DC-AC conversion
  - Generally, both DC-DC converters and DC-AC inverters use PWM control for high-speed switching.

- DC-AC conversion using an unfolding circuit
  - The DC-DC converter performs high-speed switching using PWM control, while the unfolding circuit outputs AC through low-frequency switching without PWM control.

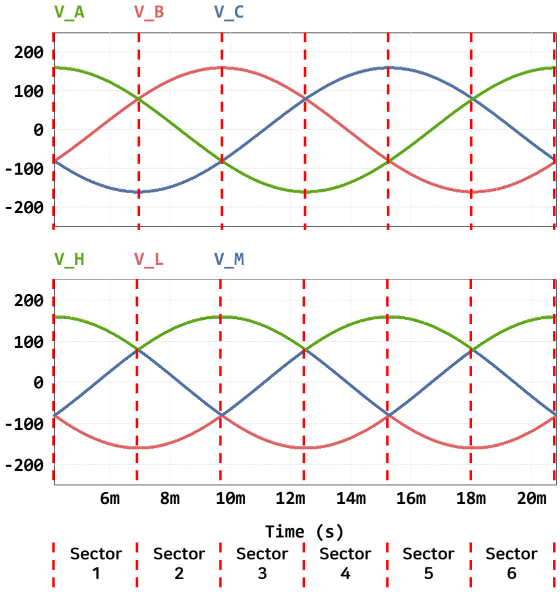
▪ Unfolding Circuit

▪ Three Phase Unfolding Circuit



<Three Phase Unfolding Circuit>

	Sector					
	1	2	3	4	5	6
SH_a	1	0	0	0	0	1
SH_b	0	0	1	1	0	0
SH_c	0	1	0	0	1	0
SM_a	0	1	1	0	0	0
SM_b	0	0	0	0	1	1
SM_c	1	0	0	1	0	0
SL_a	0	0	0	1	1	0
SL_b	1	1	0	0	0	0
SL_c	0	0	1	0	0	1



- 3 switches per phase (High / Mid / Low)
- One line cycle = 6 sectors × 60°

Three Phase Unfolding Inverter Operation Analysis

▪ Previous Research[1]

Voltage Control Method of Boost Integrated Bidirectional Three-Phase Inverter Based on Current Unfolding Topology

1st Tomoyuki Mannen
School of Engineering
Utsunomiya University
Utsunomiya, Japan
mannen@ieee.org

2nd N. Ha Pham
School of Electrical and Data Engineering
University of Technology, Sydney
Sydney, Australia
phamngocha@ieee.org

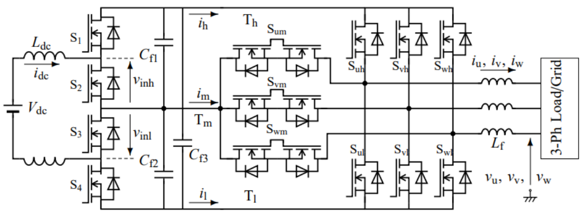


Fig. 1. Circuit diagram of the boost-integrated unfolding inverter.

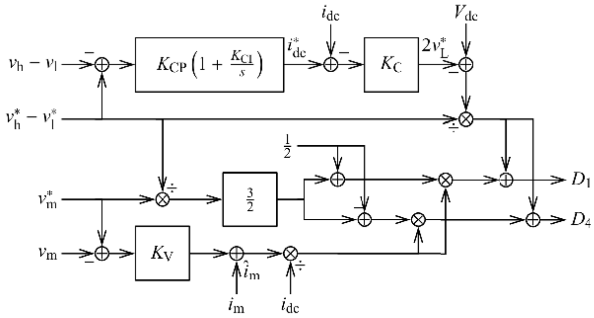


Fig. 4. Control block diagram of the three-phase battery inverter.

$$D_1 = \frac{V_{dc}-2v_L}{v_h-v_l} + \frac{v_h+2v_l}{v_h-v_l}\frac{i_m}{i_{dc}}, D_4 = \frac{V_{dc}-2v_L}{v_h-v_l} + \frac{2v_h+v_l}{v_h-v_l}\frac{i_m}{i_{dc}}$$

- **Previous Research[1] esigned the controller using only static duty-ratio relations derived from KCL and KVL, without a small-signal transfer function model.**
- **This makes control-performance optimization and stability verification difficult, and limits controller redesign under parameter variations.**

▪ Previous Research [2]

12-2017

Bidirectional Three-Phase AC-DC Power Conversion Using DC-DC Converters and a Three-Phase Unfolder

Weilun Warren Chen
Utah State University

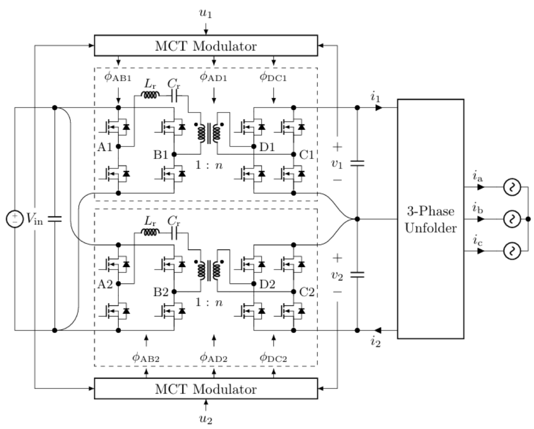
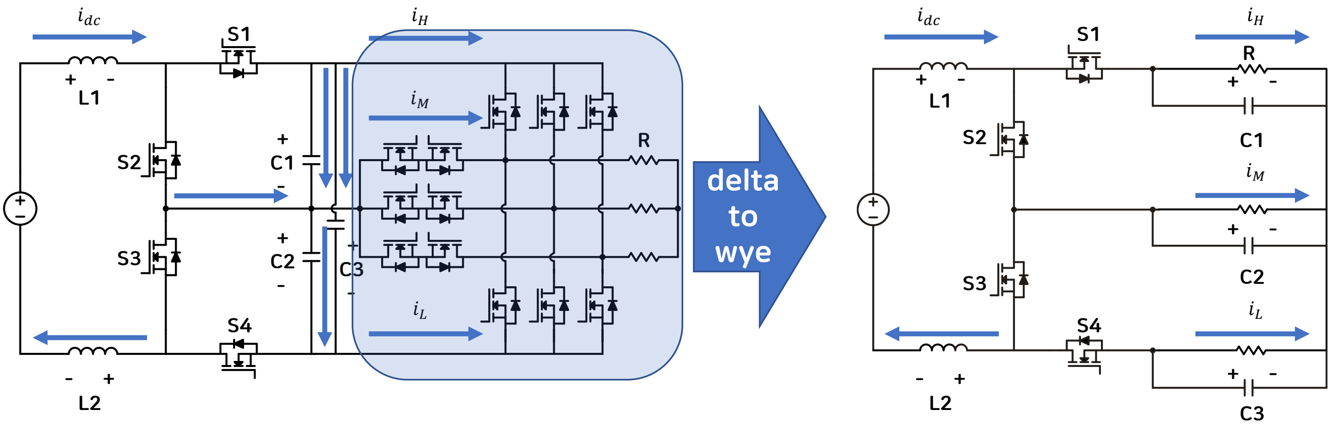


Fig. 3.11: MCT-modulated DBSRC modules with unfolder.

- **Previous Research [2] generates the input voltages of the three-phase unfolding inverter using two resonant converters.**
- **Owing to its dual-converter structure, it suffers from lower power density and higher switching losses than a single-converter approach.**
- **This approach also lacks a small-signal transfer function model, making controller design difficult.**

■ **Proposed Analysis of Three Phase Unfolding Inverter using Delta to Wye**



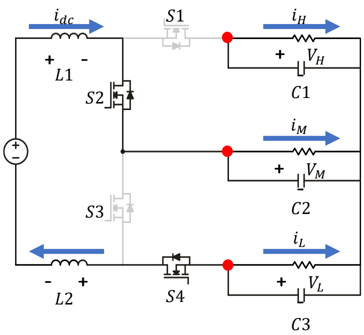
< Three Phase Unfolding Inverter Circuit>

<Simplified equivalent circuit>

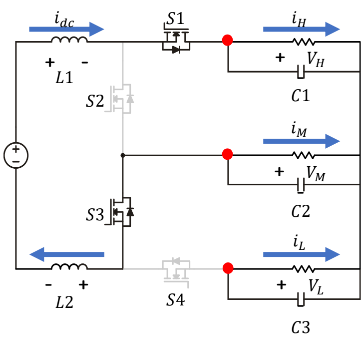
- The circuit with the T-type inverter involves many steps and is complex to analyze.
- Hence, the T-type is removed and the capacitor bank is reconfigured from delta to wye to derive the transfer function, yielding a simplified equivalent circuit.

■ Mode Analysis

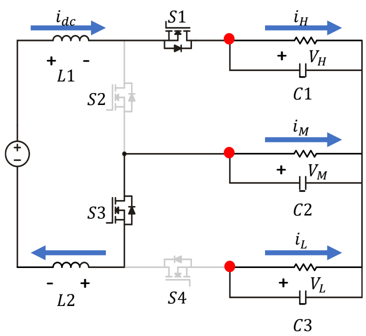
Mode1



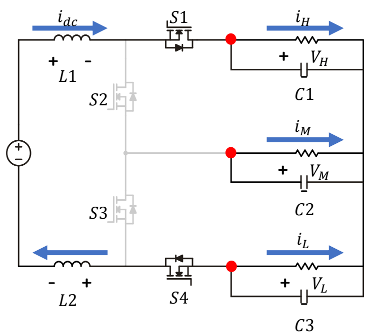
Mode2



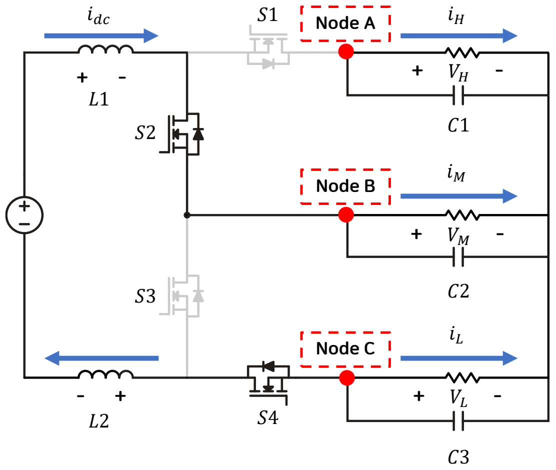
Mode3



Mode4



▪ Mode1 Model Equations



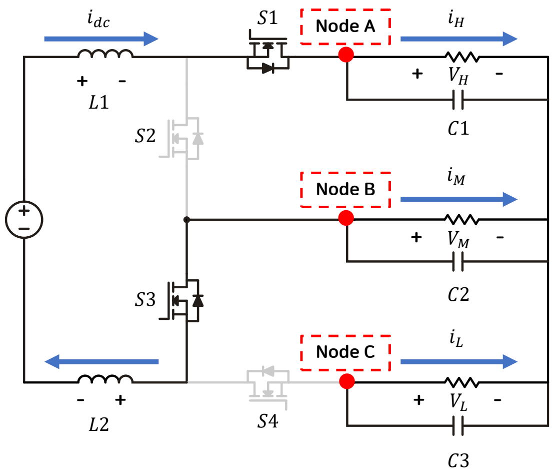
<Simplified equivalent circuit>

Conducting switches: S2, S4

- Node A (KCL)
  - $i_{C1} = -i_H$
  - $C1\frac{dV_{C1}}{dt} = -i_H = -\frac{V_H}{R}$
- Node B (KCL)
  - $i_{C2} = i_{S23} - i_M = i_{dc} - i_M$
  - $C2\frac{dV_{C2}}{dt} = i_{dc} - i_M = i_{dc} - \frac{V_M}{R}$
- Node C (KCL)
  - $i_{C3} = i_{S4} - i_L = -i_{dc} - i_L$
  - $C3\frac{dV_{C3}}{dt} = -i_{dc} - i_L = -i_{dc} - \frac{V_L}{R}$
- KVL including the voltage source
  - $2V_{L_{dc}} = V_{in} - V_M + V_L$
  - $\frac{di_{L_{dc}}}{dt} = \frac{V_{in}-V_M+V_L}{2L}$

▪ Three Phase Unfolding Inverter Operation Analysis
PEEC Laboratry

▪ Mode2 Model Equations

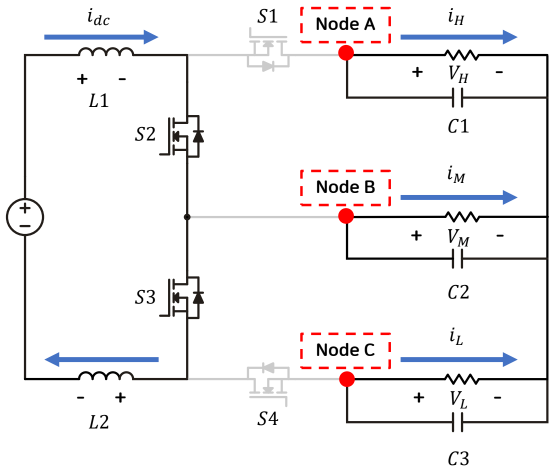


<Simplified equivalent circuit>

Conducting switches: S1, S3

- Node A (KCL)
  - $i_{C1} = i_{S1} - i_H = i_{dc} - i_H$
  - $C1\frac{dV_{C1}}{dt} = i_{dc} - i_H = i_{dc} - \frac{V_H}{R}$
- Node B (KCL)
  - $i_{C2} = i_{S23} - i_M = -i_{dc} - i_M$
  - $C2\frac{dV_{C2}}{dt} = -i_{dc} - i_M = -i_{dc} - \frac{V_M}{R}$
- Node C (KCL)
  - $i_{C3} = -i_L$
  - $C3\frac{dV_{C3}}{dt} = -i_L = -\frac{V_L}{R}$
- KVL including the voltage source
  - $2V_{L_{dc}} = V_{in} - V_H + V_M$
  - $\frac{di_{L_{dc}}}{dt} = \frac{V_{in}-V_H+V_M}{2L}$

▪ Mode3 Model Equations

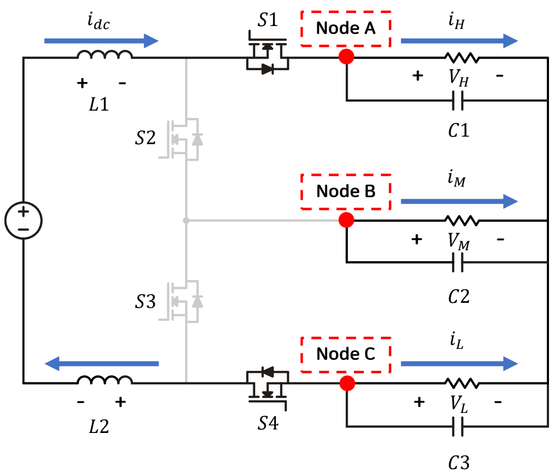


<Simplified equivalent circuit>

Conducting switches: S2, S3

- **Node A (KCL)**
  - $i_{C1} = -i_H$
  - $C1\frac{dV_{C1}}{dt} = -i_H = -\frac{V_H}{R}$
- **Node B (KCL)**
  - $i_{C2} = -i_M$
  - $C2\frac{dV_{C2}}{dt} =- -i_M = -\frac{V_M}{R}$
- **Node C (KCL)**
  - $i_{C3} = -i_L$
  - $C3\frac{dV_{C3}}{dt} = -i_L = -\frac{V_L}{R}$
- **KVL including the voltage source**
  - $2V_{L_{dc}} = V_{in}$
  - $\frac{di_{L_{dc}}}{dt} = \frac{V_{in}}{2L}$

▪ Mode4 Model Equations

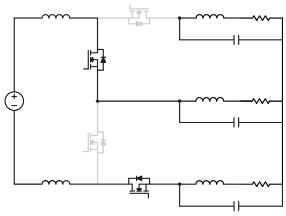


<Simplified equivalent circuit>

Conducting switches: S1, S4

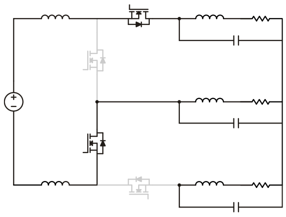
- **Node A (KCL)**
  - $i_{C1} = i_{S1} - i_H = i_{dc} - i_H$
  - $C1\frac{dV_{C1}}{dt} = i_{dc} - i_H = i_{dc} - \frac{V_H}{R}$
- **Node B (KCL)**
  - $i_{C2} = -i_M$
  - $C2\frac{dV_{C2}}{dt} =- -i_M = -\frac{V_M}{R}$
- **Node C (KCL)**
  - $i_{C3} = i_{S4} - i_L = -i_{dc} - i_L$
  - $C3\frac{dV_{C3}}{dt} = -i_{dc} - i_L = -i_{dc} - \frac{V_L}{R}$
- **KVL including the voltage source**
  - $2V_{L_{dc}} = V_{in} - V_H + V_L$
  - $\frac{di_{L_{dc}}}{dt} = \frac{V_{in}-V_H+V_L}{2L}$

▪ **State Equations by Step**



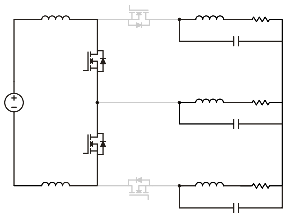
$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \\ \frac{dv_{C3}}{dt} \\ \frac{di_{L_{dc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2R} & 0 & \frac{1}{C2} \\ 0 & 0 & -\frac{1}{C_3R} & -\frac{1}{C_3} \\ 0 & -\frac{1}{2L} & \frac{1}{2L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_{C3} \\ i_{L_{dc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2L} \end{bmatrix} v_{in}$$

$< Step1 >$ $< A_{step1} >$ $< B_{step1} >$



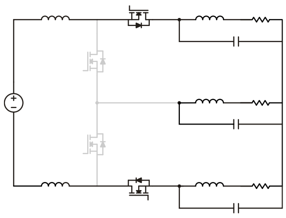
$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \\ \frac{dv_{C3}}{dt} \\ \frac{di_{L_{dc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & 0 & \frac{1}{C1} \\ 0 & -\frac{1}{C_2R} & 0 & -\frac{1}{C_2} \\ 0 & 0 & -\frac{1}{C_3R} & 0 \\ -\frac{1}{2L} & \frac{1}{2L} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_{C3} \\ i_{L_{dc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2L} \end{bmatrix} v_{in}$$

$< Step2 >$ $< A_{step2} >$ $< B_{step2} >$



$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \\ \frac{dv_{C3}}{dt} \\ \frac{di_{L_{dc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2R} & 0 & 0 \\ 0 & 0 & -\frac{1}{C_3R} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_{C3} \\ i_{L_{dc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2L} \end{bmatrix} v_{in}$$

$< Step3 >$ $< A_{step3} >$ $< B_{step3} >$

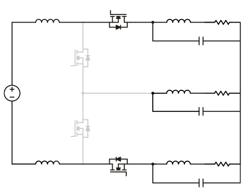


$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \\ \frac{dv_{C3}}{dt} \\ \frac{di_{L_{dc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & 0 & \frac{1}{C1} \\ 0 & -\frac{1}{C_2R} & 0 & 0 \\ 0 & 0 & -\frac{1}{C_3R} & -\frac{1}{C_3} \\ -\frac{1}{2L} & 0 & \frac{1}{2L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_{C3} \\ i_{L_{dc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2L} \end{bmatrix} v_{in}$$

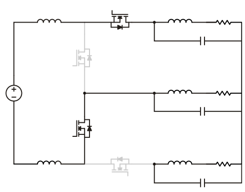
$< Step4 >$ $< A_{step4} >$ $< B_{step4} >$

▪ **State-Space Averaging of the A Matrix by Mode**

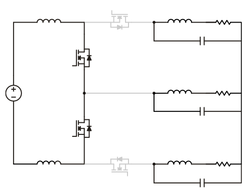
$mode1$



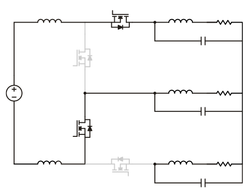
$D4$



$(D1 - D4)/2$

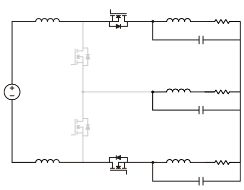


$1 - D1$

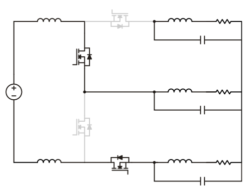


$(D1 - D4)/2$

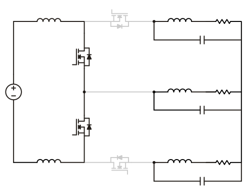
$mode2$



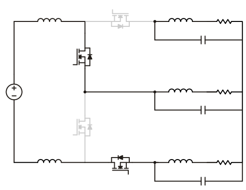
$D1$



$(D4 - D1)/2$



$1 - D4$

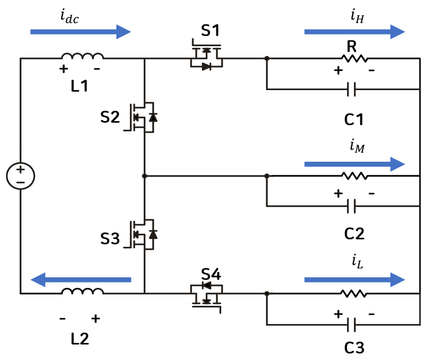


$(D4 - D1)/2$

- 4 steps → grouped into 2 operating modes
- Mode classification by capacitor voltages
- Both modes yield the same averaged A matrix
- A_{AVg}

$$\begin{bmatrix} -\frac{1}{C_1R} & 0 & 0 & \frac{D1}{C_1} \\ 0 & -\frac{1}{C_2R} & 0 & \frac{D4 - D1}{C2} \\ 0 & 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3} \\ -\frac{D1}{2L} & -\frac{D1 - D4}{2L} & \frac{D4}{2L} & 0 \end{bmatrix}$$

Voltage Conversion Ratio

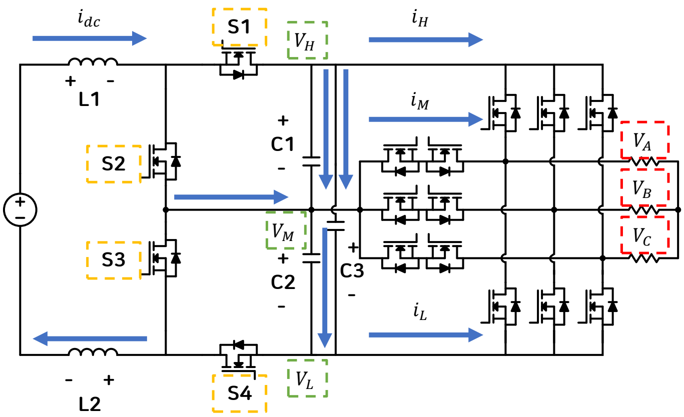


<Simplified equivalent circuit>

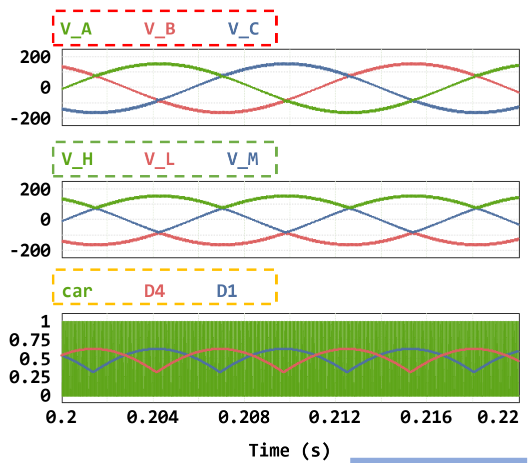
- $\frac{dv_{C1}}{dt} = -\frac{v_H}{C_1 R} + \frac{D1}{C_1} i_{dc}$
- $\frac{dv_{C2}}{dt} = -\frac{v_M}{C_2 R} + \frac{D4-D1}{C2} i_{dc}$
- $\frac{dv_{C3}}{dt} = -\frac{v_L}{C_3 R} - \frac{D4}{C_3} i_{dc}$
- $\frac{di_{L_{dc}}}{dt} = \frac{v_{in}}{2L} - \frac{D1}{2L} v_H - \frac{D1-D4}{2L} v_M + \frac{D4}{2L} v_L$

- One-cycle averaging → steady-state operating point
  - $0 = -\frac{V_H}{R} + D1 * I_{dc}$
  - $0 = -\frac{V_M}{R} + (D4 - D1) * I_{dc}$
  - $0 = -\frac{V_L}{R} - D4 * I_{dc}$
  - $0 = V_{in} - D1 * V_H - (D1 - D4)V_M + D4V_L \cdots (a)$
- Organize into V_H, V_M, V_L
  - $V_H = R * D1 * I_{dc} \cdots (1)$
  - $V_M = R * (D4 - D1) * I_{dc} \cdots (2)$
  - $V_L = -R * D4 * I_{dc} \cdots (3)$
- Substitute (1), (2), and (3) into (a)
  - $I_{dc} = \frac{V_{in}}{R*(D1^2+(D1-D4)^2+D4^2)} \cdots (4)$
- Substitute (4) into (1), (2), and (3) respectively
  - $\frac{V_H}{V_{in}} = \frac{D1}{D1^2+(D1-D4)^2+D4^2}$
  - $\frac{V_M}{V_{in}} = \frac{D4-D1}{D1^2+(D1-D4)^2+D4^2}$
  - $\frac{V_L}{V_{in}} = -\frac{D4}{D1^2+(D1-D4)^2+D4^2}$

Open-Loop Simulation Result / THD = 2 %



< Three Phase Unfolding Inverter Circuit>



- Open-loop duty D computed from the voltage conversion ratio (Slide 15)
- Output voltage THD ≈ 2 %

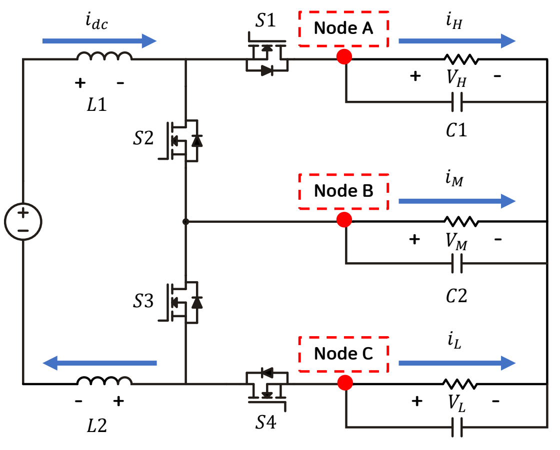
- $\frac{V_H}{V_{in}} = \frac{D1}{D1^2+(D1-D4)^2+D4^2}$
- $\frac{V_M}{V_{in}} = \frac{D4-D1}{D1^2+(D1-D4)^2+D4^2}$
- $\frac{V_L}{V_{in}} = -\frac{D4}{D1^2+(D1-D4)^2+D4^2}$

Parameter	
C_1	$1\mu F$
C_2	$1\mu F$
C_3	$1\mu F$
f_{sw}	$60kHz$

Three Phase Unfolding Inverter Transfer Function Derivation

▪ Three Phase Unfolding Inverter Transfer Function Derivation

▪ Small-Signal Modeling of the Three-Phase Unfolding Inverter



<Simplified equivalent circuit>

- 4×4 model: too complex for tractable analysis
- V_{C2} can be expressed via V_{C1} and V_{C3}
- Reduced to a 3×3 model for transfer-function derivation

- **State averaging equation**
- $$\begin{bmatrix}\frac{dv_{C1}}{dt}\\ \frac{dv_{C3}}{dt}\\ \frac{di_{L_{dc}}}{dt}\end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1}\\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3}\\ \frac{D4-2*D1}{2L} & \frac{2*D4-D1}{2L} & 0\end{bmatrix}\begin{bmatrix}V_{C1}\\ V_{C3}\\ i_{L_{dc}}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{2L}\end{bmatrix} v_{in}$$

- **Small-signal modeling**
- $$\begin{bmatrix}\frac{d(V_{C1}+\widehat{v_{C1}})}{dt}\\ \frac{d(V_{C3}+\widehat{v_{C3}})}{dt}\\ \frac{d(I_{Ldc}+\widehat{i_{Ldc}})}{dt}\end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1+\widehat{d1}}{C_1}\\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3}\\ \frac{D4-2*(D1+\widehat{d1})}{2L} & \frac{2*D4-(D1+\widehat{d1})}{2L} & 0\end{bmatrix}\begin{bmatrix}V_{C1}+\widehat{v_{C1}}\\ V_{C3}+\widehat{v_{C3}}\\ I_{Ldc}+\widehat{i_{Ldc}}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{2L}\end{bmatrix} v_{in}$$
- $$\begin{bmatrix}\frac{d(V_{C1}+\widehat{v_{C1}})}{dt}\\ \frac{d(V_{C3}+\widehat{v_{C3}})}{dt}\\ \frac{d(I_{Ldc}+\widehat{i_{Ldc}})}{dt}\end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1}\\ 0 & -\frac{1}{C_3R} & -\frac{D4+\widehat{d4}}{C_3}\\ \frac{D4+\widehat{d4}-2*D1}{2L} & \frac{2*(D4+\widehat{d4})-D1}{2L} & 0\end{bmatrix}\begin{bmatrix}V_{C1}+\widehat{v_{C1}}\\ V_{C3}+\widehat{v_{C3}}\\ I_{Ldc}+\widehat{i_{Ldc}}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{2L}\end{bmatrix} v_{in}$$

▪ Small-Signal Modeling of the Three-Phase Unfolding Inverter

- **D1 reference small signal model**

$$\begin{bmatrix} \frac{d(V_{C1}+\widehat{v_{C1}})}{dt} \\ \frac{d(V_{C3}+\widehat{v_{C3}})}{dt} \\ \frac{d(I_{Ldc}+\widehat{i_{Ldc}})}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1+\widehat{d1}}{C_1} \\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3} \\ \frac{D4-2*(D1+\widehat{d1})}{2L} & \frac{2*D4-(D1+\widehat{d1})}{2L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1}+\widehat{v_{C1}} \\ V_{C3}+\widehat{v_{C3}} \\ I_{Ldc}+\widehat{i_{Ldc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2L} \end{bmatrix} v_{in}$$

- $\frac{d(\widehat{v_{C1}})}{dt} = \left(-\frac{1}{C_1R}\right) * (\widehat{v_{C1}}) + \left(\frac{\widehat{d1}}{C_1}\right) * (I_{Ldc}) + \left(\frac{D1}{C_1}\right) * (\widehat{i_{Ldc}})$
- $\frac{d(\widehat{v_{C3}})}{dt} = \left(-\frac{1}{C_3R}\right) * (\widehat{v_{C3}}) + (-\frac{D4}{C_3}) * (\widehat{i_{Ldc}})$
- $\frac{d(\widehat{i_{Ldc}})}{dt} = \left(-2\frac{\widehat{d1}}{2L}\right) * (V_{C1}) + \left(\frac{D4-2*D1}{2L}\right) * (\widehat{v_{C1}}) + \left(-\frac{\widehat{d1}}{2L}\right) * (V_{C3}) + \left(\frac{2*D4-D1}{2L}\right) * (\widehat{v_{C3}})$

$$\begin{bmatrix} \frac{d\widehat{v_{C1}}}{dt} \\ \frac{d\widehat{v_{C3}}}{dt} \\ \frac{d\widehat{i_{Ldc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1} \\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3} \\ \frac{D4-2*D1}{2L} & \frac{2*D4-D1}{2L} & 0 \end{bmatrix} \begin{bmatrix} \widehat{v_{C1}} \\ \widehat{v_{C3}} \\ \widehat{i_{Ldc}} \end{bmatrix} + \begin{bmatrix} \frac{I_{Ldc}}{C_1} \\ 0 \\ -\frac{2V_{C1}+V_{C3}}{2L} \end{bmatrix} \widehat{d1}$$

- **D4 reference small signal model**

$$\begin{bmatrix} \frac{d(V_{C1}+\widehat{v_{C1}})}{dt} \\ \frac{d(V_{C3}+\widehat{v_{C3}})}{dt} \\ \frac{d(I_{Ldc}+\widehat{i_{Ldc}})}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1} \\ 0 & -\frac{1}{C_3R} & -\frac{D4+\widehat{d4}}{C_3} \\ \frac{D4+\widehat{d4}-2*D1}{2L} & \frac{2*(D4+\widehat{d4})-D1}{2L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1}+\widehat{v_{C1}} \\ V_{C3}+\widehat{v_{C3}} \\ I_{Ldc}+\widehat{i_{Ldc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2L} \end{bmatrix} v_{in}$$

- $\frac{d(\widehat{v_{C1}})}{dt} = \left(-\frac{1}{C_1R}\right) * (\widehat{v_{C1}}) + \left(\frac{D1}{C_1}\right) * (\widehat{i_{Ldc}})$
- $\frac{d(\widehat{v_{C3}})}{dt} = \left(-\frac{1}{C_3R}\right) * (\widehat{v_{C3}}) + \left(-\frac{\widehat{d4}}{C_3}\right) * (I_{Ldc}) + (-\frac{D4}{C_3}) * (\widehat{i_{Ldc}})$
- $\frac{d(\widehat{i_{Ldc}})}{dt} = \left(\frac{\widehat{d4}}{2L}\right) * (V_{C1}) + \left(\frac{D4-2*D1}{2L}\right) * (\widehat{v_{C1}}) + \left(2\frac{\widehat{d4}}{2L}\right) * (V_{C3}) + \left(\frac{2*D4-D1}{2L}\right) * (\widehat{v_{C3}})$

$$\begin{bmatrix} \frac{d\widehat{v_{C1}}}{dt} \\ \frac{d\widehat{v_{C3}}}{dt} \\ \frac{d\widehat{i_{Ldc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1} \\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3} \\ \frac{D4-2*D1}{2L} & \frac{2*D4-D1}{2L} & 0 \end{bmatrix} \begin{bmatrix} \widehat{v_{C1}} \\ \widehat{v_{C3}} \\ \widehat{i_{Ldc}} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\mathrm{I_{Ldc}}}{C_3} \\ \frac{V_{C1}+2V_{C3}}{2L} \end{bmatrix} \widehat{d4}$$

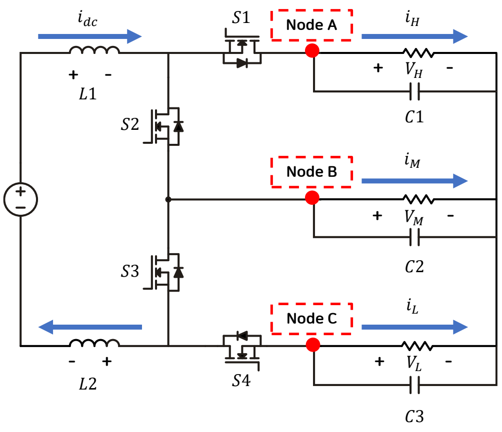
▪ Derivation of the Transfer Function with Respect to D₁

$$\begin{bmatrix} \frac{d\widehat{v_{C1}}}{dt} \\ \frac{d\widehat{v_{C3}}}{dt} \\ \frac{d\widehat{i_{Ldc}}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1} \\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3} \\ \frac{D4-2*D1}{2L} & \frac{2*D4-D1}{2L} & 0 \end{bmatrix} \begin{bmatrix} \widehat{v_{C1}} \\ \widehat{v_{C3}} \\ \widehat{i_{Ldc}} \end{bmatrix} + \begin{bmatrix} \frac{I_{Ldc}}{C_1} \\ 0 \\ -\frac{2V_{C1}+V_{C3}}{2L} \end{bmatrix} \widehat{d1}, \; A = \begin{bmatrix} -\frac{1}{C_1R} & 0 & \frac{D1}{C_1} \\ 0 & -\frac{1}{C_3R} & -\frac{D4}{C_3} \\ \frac{D4-2*D1}{2L} & \frac{2*D4-D1}{2L} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{I_{Ldc}}{C_1} \\ 0 \\ -\frac{2V_{C1}+V_{C3}}{2L} \end{bmatrix}, sI-A = \begin{bmatrix} s+\frac{1}{C_1R} & 0 & -\frac{D1}{C_1} \\ 0 & s+\frac{1}{C_3R} & \frac{D4}{C_3} \\ -\frac{D4-2*D1}{2L} & -\frac{2*D4-D1}{2L} & 0 \end{bmatrix}$$

- $(sI-A)X(s) = BU(s)$ **-> Calculate transfer function using Cramer's formula**
- $N_{vc1d}(s) = \begin{vmatrix} \frac{I_{Ldc}}{C_1} & 0 & -\frac{D1}{C_1} \\ 0 & s+\frac{1}{C_3R} & \frac{D4}{C_3} \\ -\frac{2V_{C1}+V_{C3}}{2L} & -\frac{2*D4-D1}{2L} & 0 \end{vmatrix} = (\frac{I_L}{C1}) * (s^2 + \frac{s}{C3*R} + \frac{2*D4^2-D1*D4}{2*L*C3}) - (\frac{D1}{C1}) * (\frac{2*V_{C1}+V_{C3}}{2*L}) * (s + \frac{1}{C3*R})$
- $\det(sI-A) = \Delta s = s^3 + \left(\frac{1}{C1*R} + \frac{1}{C3*R}\right) * s^2 + \left(\frac{1}{C1*C3*R^2} - \frac{(D1*D4-2*D1^2)}{2*L*C1} - \frac{(D1*D4-2*D4^2)}{2*L*C3}\right) * s + \frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}$
- $\frac{\mathrm{V}_{C1}(s)}{\mathrm{d}_1(s)} = \frac{N_{vc1d}(s)}{\Delta s} = \frac{(\frac{I_L}{C1})*(s^2+\frac{s}{C3*R}+\frac{2*D4^2-D1*D4}{2*L*C3})-(\frac{D1}{C1})*(\frac{2*V_{C1}+V_{C3}}{2*L})*(s+\frac{1}{C3*R})}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$

▪ Three Phase Unfolding Inverter overall transfer function

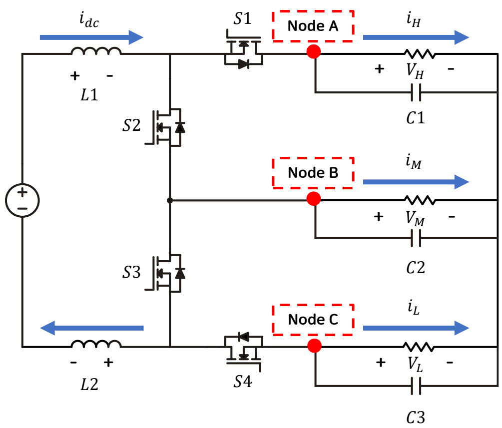
- $$\frac{V_{C1}(s)}{d_1(s)}=\frac{\frac{I_L}{C1}*s^2+\left(\frac{I_{Ldc}}{C1*C3*R}-\frac{D1(2V_{C1}+V_{C3})}{2*C1*L}\right)s+\frac{2D4^2*I_{Ldc}-D1*D4*I_{Ldc}}{2*C1*C3*L}-\frac{2V_{C1}*D1+V_{C3}*D1}{2*C1*C3*L*R}}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$$
- $$\frac{V_{C3}(s)}{d_1(s)}=\frac{\frac{D4(2V_{C1}+V_{C3})}{2*C3*L}*s+\frac{D4(2V_{C1}+V_{C3})}{2*C1*C3*L*R}-\frac{D4*I_{Ldc}(D_4-2D_1)}{2*C1*C3L}}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$$
- $$\frac{V_{C1}(s)}{d_4(s)}=\frac{\frac{D1(V_{C1}+2V_{C3})}{2*C1*L}*s+\frac{D1(V_{C1}+2V_{C3})}{2*C1*C3*L*R}-\frac{D1*I_{Ldc}(2D_4-D_1)}{2*C1*C3L}}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$$
- $$\frac{V_{C3}(s)}{d_4(s)}=\frac{-\frac{I_L}{C3}*s^2+\left(-\frac{I_{Ldc}}{C1*C3*R}-\frac{D4(V_{C1}+2V_{C3})}{2*C3*L}\right)s+\frac{D1*D4*I_{Ldc}-2D1^2*I_{Ldc}}{2*C1*C3*L}-\frac{V_{C1}*D4+2V_{C3}*D4}{2*C1*C3*L*R}}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$$
- $$\frac{I_{Ldc}(s)}{d_1(s)}=\frac{-\left(\frac{2V_{C1}+V_{C3}}{2L}\right)*s^2+\left(\frac{(D4-2D1)I_{Ldc}}{2*C1*L}-\frac{2V_{C1}+V_{C3}}{2L}\left(\frac{1}{C1R}+\frac{1}{C_3R}\right)\right)*s+\frac{(D4-2D1)I_{Ldc}}{2*C1*C3*L*R}-\frac{2V_{C1}+V_{C3}}{2*C1*C3*L*R^2}}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$$
- $$\frac{I_{Ldc}(s)}{d_4(s)}=\frac{\left(\frac{V_{C1}+2V_{C3}}{2L}\right)*s^2+\left(-\frac{(2D4-D1)I_{Ldc}}{2*C3*L}-\frac{V_{C1}+2V_{C3}}{2L}\left(\frac{1}{C1R}+\frac{1}{C_3R}\right)\right)*s-\frac{(2D4-D1)I_{Ldc}}{2*C1*C3*L*R}+\frac{V_{C1}+2V_{C3}}{2*C1*C3*L*R^2}}{s^3+\left(\frac{1}{C1*R}+\frac{1}{C3*R}\right)*s^2+\left(\frac{1}{C1*C3*R^2}-\frac{(D1*D4-2*D1^2)}{2*L*C1}-\frac{(D1*D4-2*D4^2)}{2*L*C3}\right)*s+\frac{(D1^2-D1*D4+D4^2)}{C1*C3*R*L}}$$



<Simplified equivalent circuit>

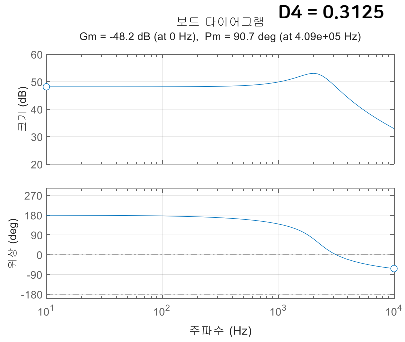
- A total of six transfer functions are derived.

▪ Bode plot of the Three Phase Unfolding Inverter



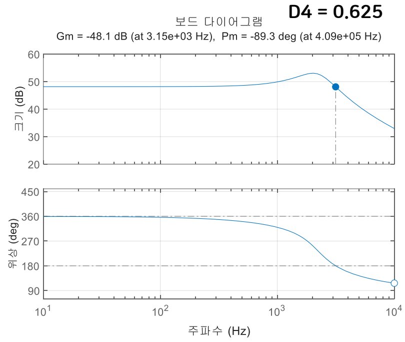
<Simplified equivalent circuit>

D1 =0.625
D4 = 0.3125



<$V_{C1}(s)/d_1(s)$ Bode plot >

D1 = 0.3125
D4 = 0.625



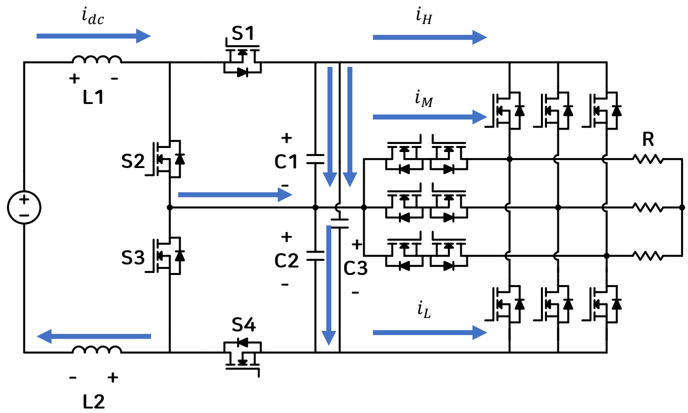
<$V_{C3}(s)/d_4(s)$ Bode plot >

		$\frac{V_{C1}(s)}{d_1(s)}$	$\frac{V_{C3}(s)}{d_4(s)}$
100Hz (Matlab)	magnitude	48dB	48dB
	phase	177°	357°
1kHz (Matlab)	magnitude	49.5dB	49.5dB
	phase	139°	319°

- Bode plot of the simplified equivalent circuit transfer function via MATLAB

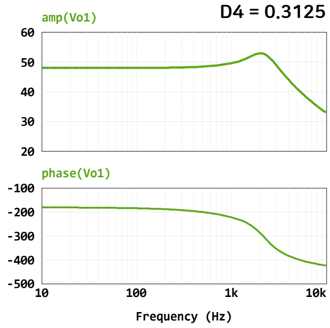
▪ Three Phase Unfolding Inverter Transfer Function Derivation

▪ Bode plot of the Three Phase Unfolding Inverter



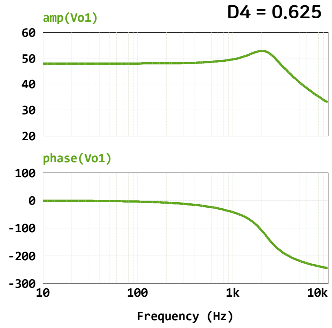
< Three Phase Unfolding Inverter Circuit>

D1 =0.625
D4 = 0.3125



< $V_H(s)\ /d_1(s)$ Bode plot >

D1 = 0.3125
D4 = 0.625



<$V_L(s)\ /d_4(s)$ Bode plot >

		$\frac{V_{C1}(s)}{d_1(s)}$	$\frac{V_{C3}(s)}{d_4(s)}$
100Hz	magnitude	48dB	48dB
	phase	-183°	-3°
1kHz	magnitude	49.5dB	49.5dB
	phase	-221°	-41°

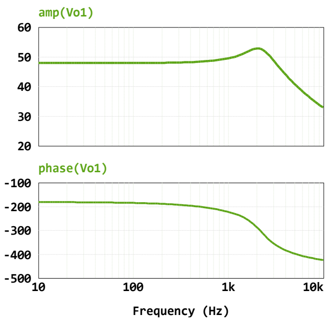
- **Bode plot of the TLBC unfolding inverter circuit via PSIM AC Sweep**

▪ Three Phase Unfolding Inverter Transfer Function Derivation

▪ Comparison of MATLAB and PSIM AC Sweep

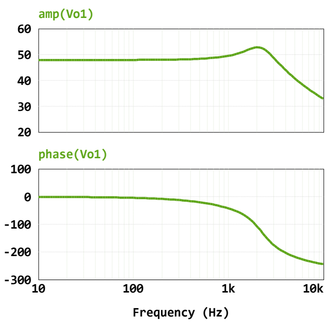
$$\frac{V_H(s)}{d_1(s)} = \frac{V_{C1}(s)}{d_1(s)}$$

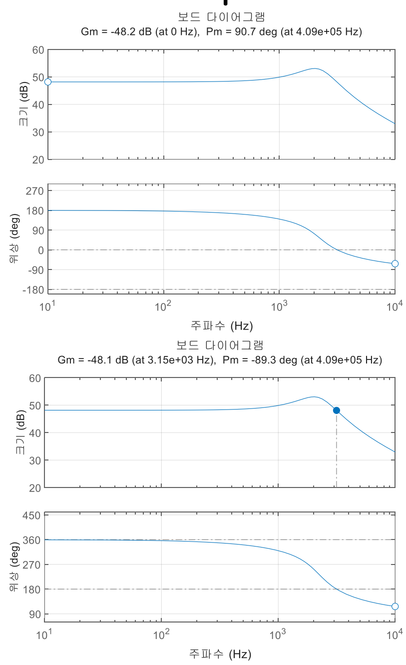
D1 =0.625
D4 = 0.3125



$$\frac{V_L(s)}{d_4(s)} = \frac{V_{C3}(s)}{d_4(s)}$$

D1 = 0.3125
D4 = 0.625





- **Comparison of Transfer Function and PSIM AC Sweep**
- **The magnitude (dB) matches.**
- **The phase agrees within 360° (i.e., effectively identical).**

		$\frac{V_{C1}(s)}{d_1(s)}$	$\frac{V_{C3}(s)}{d_4(s)}$
100Hz (PSIM)	magnitude	48dB	48dB
	phase	-183°	-3°
100Hz (Matlab)	magnitude	48dB	48dB
	phase	177°	357°
1kHz (PSIM)	magnitude	49.5dB	49.5dB
	phase	-221°	-41°
1kHz (Matlab)	magnitude	49.5dB	49.5dB
	phase	139°	319°

Conclusion

PEEC Laboratry

① Summary

- Operation analysis of the three-phase unfolding inverter was performed by analyzing 4 switching steps and 2 averaged modes.
- The equivalent circuit was simplified by removing the T-type stage and converting the capacitor bank from delta to wye, enabling tractable analysis.
- Small-signal modeling via state-space averaging yielded six transfer functions with respect to the duty ratios.
- The model was validated against PSIM AC-sweep: magnitude in dB matches and phase matches within a 360° offset.

② Significance

- Previous Research[1] lacks a transfer-function model (only static KCL/KVL relations), and Previous Research[2] requires a dual-converter input stage.
- This work derives the missing MIMO small-signal transfer function on a single-converter input stage (TLBC).
- The derived transfer function makes stability and bandwidth analytically predictable, providing a clear basis for systematic controller design.

③ Future Work

- Closed-loop voltage / current controller design based on the derived MIMO plant.
- Hardware experimental verification with a prototype.

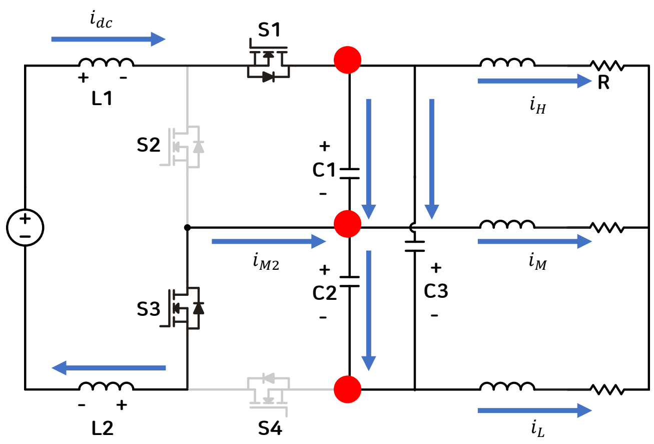
Q & A

Appendix

▪ Three Phase Unfolding Inverter Operation Analysis

PEEC Laboratry

▪ Step1



- $V_M = -V_H - V_L$
- $V_{C1} = V_H - V_M$
- $V_{C2} = V_M - V_L$
- $V_{C3} = V_H - V_L$

- $V_M = \frac{V_{C2} - V_{C1}}{3}$
- $V_H = \frac{2V_{C1} + V_{C2}}{3}$
- $V_L = -\frac{V_{C1} + 2V_{C2}}{3}$

- $i_{C1} + i_{C3} = i_{dc} - i_H$
- $C1\frac{dV_{C1}}{dt} + C3\frac{dV_{C3}}{dt} = i_{dc} - i_H = i_{dc} - \frac{V_H}{R} = i_{dc} - \frac{2V_{C1} + V_{C2}}{3R}$

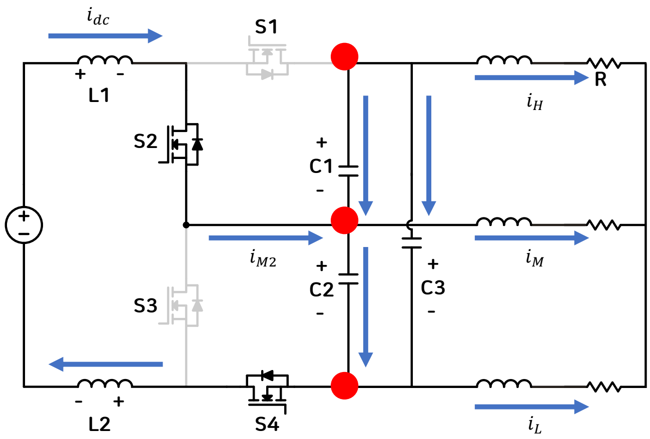
- $i_{C1} - i_{C2} = i_M - i_{M2}$
- $C1\frac{dV_{C1}}{dt} - C2\frac{dV_{C2}}{dt} = i_M - i_{M2}$

- $i_{C2} + i_{C3} = i_L$
- $C2\frac{dV_{C2}}{dt} + C3\frac{dV_{C3}}{dt} = i_L = \frac{V_L}{R} = -\frac{V_{C1} + 2V_{C2}}{3R}$

- $2V_L = V_{in} - V_{C1}$
- $\frac{di_L}{dt} = \frac{V_{in} - V_{C1}}{2L}$

▪ Three Phase Unfolding Inverter Operation Analysis

▪ Step2



- $V_M = -V_H - V_L$
- $V_{C1} = V_H - V_M$
- $V_{C2} = V_M - V_L$
- $V_{C3} = V_H - V_L$

→

- $V_M = \frac{V_{C2}-V_{C1}}{3}$
- $V_H = \frac{2V_{C1}+V_{C2}}{3}$
- $V_L = -\frac{V_{C1}+2V_{C2}}{3}$

- $i_{C1} + i_{C3} = -i_H$
- $C1\frac{dV_{C1}}{dt} + C3\frac{dV_{C3}}{dt} = -i_H = -\frac{V_H}{R} = -\frac{2V_{C1}+V_{C2}}{3R}$

- $i_{C1} - i_{C2} = i_M - i_{M2}$
- $C1\frac{dV_{C1}}{dt} - C2\frac{dV_{C2}}{dt} = i_M - i_{M2}$

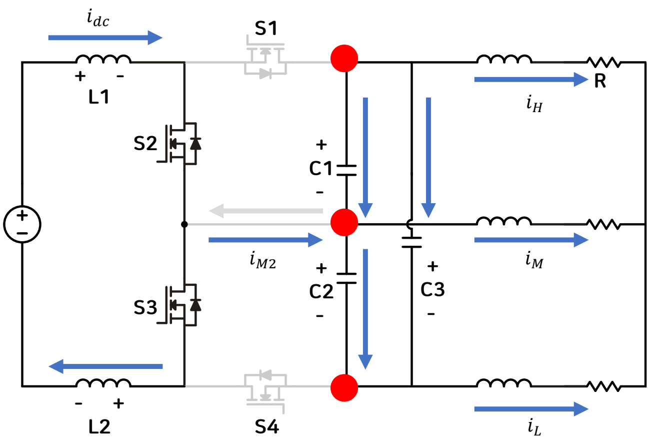
- $i_{C2} + i_{C3} = i_{dc} + i_L$
- $C2\frac{dV_{C2}}{dt} + C3\frac{dV_{C3}}{dt} = i_{dc} + i_L = i_{dc} + \frac{V_L}{R} = i_{dc} - \frac{V_{C1}+2V_{C2}}{3R}$

- $2V_L = V_{in} - V_{C2}$
- $\frac{di_L}{dt} = \frac{V_{in}-V_{C2}}{2L}$

▪ Three Phase Unfolding Inverter Operation Analysis

PEEC Laboratry

▪ Step3



- $V_M = -V_H - V_L$
- $V_{C1} = V_H - V_M$
- $V_{C2} = V_M - V_L$
- $V_{C3} = V_H - V_L$

→

- $V_M = \frac{V_{C2}-V_{C1}}{3}$
- $V_H = \frac{2V_{C1}+V_{C2}}{3}$
- $V_L = -\frac{V_{C1}+2V_{C2}}{3}$

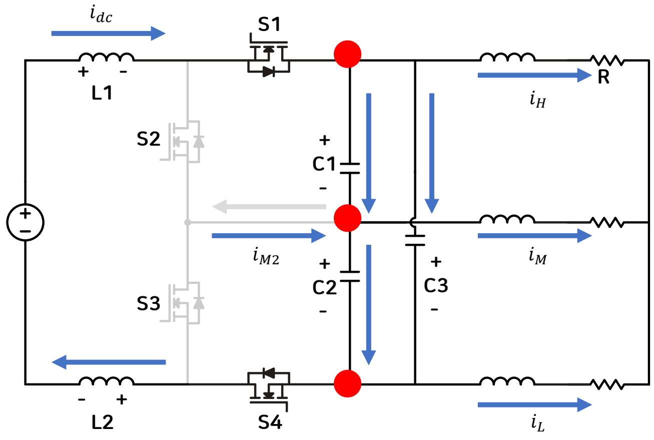
- $i_{C1} + i_{C3} = -i_H$
- $C1\frac{dV_{C1}}{dt} + C3\frac{dV_{C3}}{dt} = -i_H = -\frac{V_H}{R} = -\frac{2V_{C1}+V_{C2}}{3R}$

- $i_{C1} - i_{C2} = i_M$
- $C1\frac{dV_{C1}}{dt} - C2\frac{dV_{C2}}{dt} = i_M$

- $i_{C2} + i_{C3} = i_L$
- $C2\frac{dV_{C2}}{dt} + C3\frac{dV_{C3}}{dt} = i_L = \frac{V_L}{R} = -\frac{V_{C1}+2V_{C2}}{3R}$

- $2V_L = V_{in}$
- $\frac{di_L}{dt} = \frac{V_{in}}{2L}$

▪ Step4



- $V_M = -V_H - V_L$
- $V_{C1} = V_H - V_M$
- $V_{C2} = V_M - V_L$
- $V_{C3} = V_H - V_L$

- $V_M = \frac{V_{C2} - V_{C1}}{3}$
- $V_H = \frac{2V_{C1} + V_{C2}}{3}$
- $V_L = -\frac{V_{C1} + 2V_{C2}}{3}$

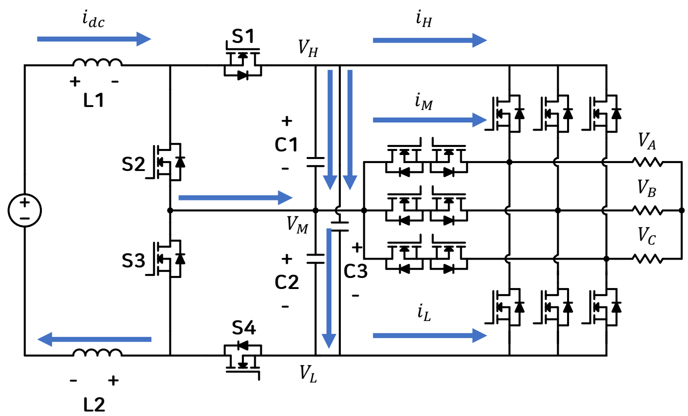
- $i_{C1} + i_{C3} = i_{dc} - i_H$
- $C1\frac{dV_{C1}}{dt} + C3\frac{dV_{C3}}{dt} = i_{dc} - i_H = i_{dc} - \frac{V_H}{R} = i_{dc} - \frac{2V_{C1} + V_{C2}}{3R}$

- $i_{C1} - i_{C2} = i_M$
- $C1\frac{dV_{C1}}{dt} - C2\frac{dV_{C2}}{dt} = i_M$

- $i_{C2} + i_{C3} = i_{dc} + i_L$
- $C2\frac{dV_{C2}}{dt} + C3\frac{dV_{C3}}{dt} = i_{dc} + i_L = i_{dc} + \frac{V_L}{R} = i_{dc} - \frac{V_{C1} + 2V_{C2}}{3R}$

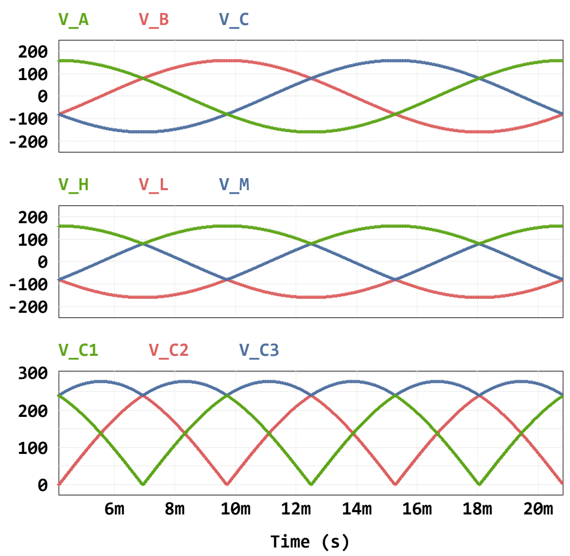
- $2V_L = V_{in} - V_{C3} = V_{in} - V_H + V_L$
- $\frac{di_L}{dt} = \frac{V_{in}}{2L} - \frac{2V_{C1} + V_{C2}}{6L} - \frac{V_{C1} + 2V_{C2}}{6L}$

▪ Key operating waveforms of the Three Phase Unfolding Inverter



< Three Phase Unfolding Inverter Circuit>

- Key operating waveforms of the three-phase unfolding inverter.
- Each capacitor voltage is shaped by the TLBC converter, and the three-phase unfolding circuit alternates its polarity to generate the three-phase AC output.



<Key operating waveforms>